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INTENSIVE DRYING OF AN INFINITE PLATE

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Abstract—Kinetics of drying mechanism is studied for the capillary porous body. Local values of temperature, moisture content and pressure are evaluated analytically under the most general type of boundary conditions. The novelty of the work lies in consideration of the generalised surface conditions which includes all types of the interaction law between the moist body and the surrounding. Analytical result indicates that the process is intensified by the filtrational drying.

NOMENCLATURE

a, coefficient of diffusivity;
A_i, B_i $(i = 1, 2)$, C_1, dimensionless known thermophysical coefficients;
c, specific content;
Fo, Fourier number;
p, pressure;
r, space variable;
$2R$, plate thickness;
s, Laplace parameter;
t, time variable;
T, temperature distribution;
U, concentration of the matter;
x, dimensionless variable.

Greek symbols

λ, coefficient of conductivity;
γ, density;
ε, phase criterion;
ρ, specific heat of evaporation;
δ, Soret coefficient;
θ_i, dimensionless transfer potentials;
μ_n, characteristic root defined by equation (25);
$\phi_i(Fo)$, prescribed fluxes;
ψ_n, defined by equation (26).

Subscript

m, matter;
q, heat;
p, pressure.

Superscript

0, initial value.

Suffixes

i, j, 1, 2, 3;
n, $1, 2, \ldots, \infty$.

INTRODUCTION

IN THE process of drying, the moisture is transferred from inside the material towards the surface where it evaporates. In general, the rate of drying depends upon the rate of moisture transport from the inside of the material towards its surface. The experimental research of Lebedev [1], Maximov [4] and others has proved that the process of drying is intensified by the action of the various hydrodynamical forces. Studies of the kinetics of drying mechanism usually involve an investigation of the moisture content and temperature distribution inside the drying material. The moisture transport depends upon the gradients of moisture content, temperature and total pressure [2]. Thus, the determination of the local values of the temperature, moisture content and pressure entails the solution of the differential equations of heat, moisture and filtrational transfer.

Mikhailov [5] has determined the local values of the temperature, moisture content and pressure distribution for the basic bodies like infinite plate and sphere under convective type of boundary conditions. Toei and Okazaki [7] have also proposed a new model for convective drying mechanism of capillary porous bodies. However in a number of processes of intensive drying, the surface of the dried material does not follow the convective type of boundary conditions, i.e. the specific heat or moisture flux is not always proportional to the temperature or the moisture head because the heat-transfer coefficient or Nusselt number is a function of the moisture content of the material being dried.

In the present analysis, the local values of the transfer potentials are determined under the most general type of boundary condition. The kinetics of drying mechanism have been investigated in detail.

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FORMULATION

Moist materials are capillary porous in structure containing inert gases and water particles in all the three states. In a nonisothermal state, the local values of temperature, moisture content and pressure in a capillary porous body are determined by a set of transport equations [3]

$$c_q\gamma \frac{\partial T}{\partial t} = \lambda_q \frac{\partial^2 T}{\partial r^2} + \varepsilon\rho c_m\gamma \frac{\partial U}{\partial t} \tag{1}$$

$$c_m\gamma \frac{\partial U}{\partial t} = \lambda_m \frac{\partial^2 U}{\partial r^2} + \lambda_m\delta \frac{\partial^2 T}{\partial r^2} + \lambda_p \frac{\partial^2 p}{\partial r^2} \tag{2}$$

and

$$c_p\gamma \frac{\partial p}{\partial t} = \lambda_p \frac{\partial^2 p}{\partial r^2} - \varepsilon c_m\gamma \frac{\partial U}{\partial t} \tag{3}$$

where the first term on the RHS of equation (1) denotes the change in temperature due to heat conduction and the second term denotes the change in temperature due to phase transformation. Similarly the changes in moisture content and pressure are described by equations (2) and (3) respectively.

The most significant approximations in writing the set of equations (1)–(3) are that viscous forces are disregarded and the material properties are constant. The velocities of heat and moisture propagation are infinite, an assumption which is inconsistent with the physics of the transport phenomena, however, such an assumption does not introduce any overall error for practical purposes. Also, there is no convective transfer of heat by current of matter in the system. For simplicity, we shall transform equations (1)–(3) in the dimensionless form by introducing some non-dimensional variables:

$$x = \frac{r}{R}, \quad Fo = \frac{a_q t}{R^2}, \quad \theta_1 = \frac{T - T^0}{T^0}, \quad \theta_2 = \frac{U^0 - U}{U^0}, \quad \theta_3 = \frac{p - p^0}{p^0}$$

and Luikov's criteria for the field of matter and filtration in relation to the temperature field

$$Lu = \frac{a_m}{a_q}, \quad Lu_p = \frac{a_p}{a_q}$$

Kossovich criterion

$$Ko = c_m\rho \frac{U^0}{T^0}$$

Posnov criterion

$$Pn = \delta \frac{T^0}{U^0}$$

and Bulygin criterion

$$Bu = \rho \frac{c_p}{c_q} \frac{p^0}{T^0}.$$

The equations (1)–(3) then finally reduce to

$$\frac{\partial\theta_1}{\partial Fo} = \frac{\partial^2\theta_1}{\partial x^2} - \varepsilon Ko \frac{\partial\theta_2}{\partial Fo} \tag{4}$$

$$\frac{\partial\theta_2}{\partial Fo} = Lu \frac{\partial^2\theta_2}{\partial x^2} - LuPn \frac{\partial^2\theta_1}{\partial x^2} - Lu_p \frac{Bu}{Ko} \frac{\partial^2\theta_3}{\partial x^2} \tag{5}$$

and

$$\frac{\partial\theta_3}{\partial Fo} = Lu_p \frac{\partial^2\theta_3}{\partial x^2} + \varepsilon \frac{Ko}{Bu} \frac{\partial\theta_2}{\partial Fo} \tag{6}$$

The set of equations (4)–(6) has been solved by Luikov and Mikhailov under various boundary conditions. They have also considered a situation where the specific flux of mass varies continuously with the time. Still a more general case can be attempted by prescribing the specific fluxes of heat and mass as certain unknown functions of time, to be determined by the experiment. In this way, the following boundary conditions may be prescribed for the system

$$\frac{\partial \theta_1}{\partial x} + A_1\theta_1 + B_1\theta_2 = \phi_1(Fo) \tag{7}$$

$$\frac{\partial \theta_2}{\partial x} + A_2\frac{\partial \theta_1}{\partial x} + B_2\theta_2 + c_1\frac{\partial \theta_3}{\partial x} = \phi_2(Fo) \tag{8}$$

and

$$\theta_3 = \phi_3(Fo). \tag{9}$$

It should be noted here that the above boundary conditions describe a large class of heat- and mass-transfer phenomena including radiative heat transfer. In addition, we shall suppose that the system is symmetrical thermally and geometrically, i.e.

$$\frac{\partial \theta_i}{\partial x} = 0, \quad x = 0. \tag{10}$$

Further that the transfer potentials are distributed uniformly at the initial moment of time.

$$\theta_i(x, 0) = \theta_{i0} \quad \text{(constant)}. \tag{11}$$

SOLUTION

The solution of the set of differential equations (4)–(6) under the conditions (7)–(11) is obtained by the application of Laplace transform technique. The solution can be written under transformed form as:

$$\bar{\theta}_1 = \sum_{j=1}^{3} \frac{1}{\psi_0(s)} L_j \cosh v_j(\sqrt{s})x \tag{12}$$

$$\bar{\theta}_2 = \frac{1}{\varepsilon Ko} \sum_{j=1}^{3} \frac{1}{\psi_0(s)} L_j(1-v_j^2) \cosh v_j(\sqrt{s})x \tag{13}$$

and

$$\bar{\theta}_3 = \frac{1}{\varepsilon Bu} \sum_{j=1}^{3} \frac{1}{\psi_0(s)} \sigma_j \cosh v_j(\sqrt{s})x \tag{14}$$

where the symbols σ_j and v_j are defined as

$$\sigma_j = (1-\varepsilon)(1-v_j^2) - Luv_j^2(1-v_j^2) - \varepsilon KoPnLuv_j^2 \tag{15}$$

and

$$v_j = \sqrt{(y_j + \tfrac{1}{3}\alpha)} \tag{16}$$

where y_j are the roots of the cubic equation

$$y^3 + \pi_1 y + \pi_2 = 0,$$

$$\pi_1 = -\tfrac{1}{3}\alpha^2 + \beta, \quad \pi_2 = -\tfrac{2}{27}\alpha^3 + \tfrac{1}{3}\alpha\beta - \gamma$$

$$\alpha = 1 + (1-\varepsilon)\frac{1}{Lu} + \frac{1}{Lu_p} + \varepsilon KoPn$$

$$\beta = (1-\varepsilon)\frac{1}{Lu} + \left(1 + \varepsilon KoPn + \frac{1}{Lu}\right)\frac{1}{Lu_p}$$

and

$$\gamma = \frac{1}{LuLu_p}.$$

Further,

$$P_{ij} = \left(A_1 + \frac{1-v_j^2}{\varepsilon Ko}B_1\right)\cosh v_j(\sqrt{s}) + v_j(\sqrt{s})\sinh v_j(\sqrt{s})$$

$$P_{2j} = \frac{1-v_j^2}{\varepsilon Ko}B_2 \cosh v_j(\sqrt{s}) + \left(\frac{1-v_j^2}{\varepsilon Ko} + A_2 + \frac{\sigma_j C_1}{\varepsilon Bu}\right)v_j(\sqrt{s})\sinh v_j(\sqrt{s})$$

$$P_{3j} = \frac{\sigma_j}{\varepsilon Bu}\cosh v_j(\sqrt{s})$$

and

$$L_1 = \left(P_{22} - \frac{P_{32}}{P_{33}} P_{23}\right)\bar{\phi}_1(s) - \left(P_{12} - \frac{P_{32}}{P_{33}} P_{13}\right)\bar{\phi}_2(s) + \frac{1}{P_{33}}(P_{12}P_{23} - P_{22}P_{13})\bar{\phi}_3(s)$$

$$-L_2 = \left(P_{21} - \frac{P_{31}}{P_{33}} P_{23}\right)\bar{\phi}_1(s) - \left(P_{11} - \frac{P_{31}}{P_{33}} P_{13}\right)\bar{\phi}_2(s) + \frac{1}{P_{33}}(P_{21}P_{13} - P_{11}P_{23})\bar{\phi}_3(s)$$

$$L_3 = \frac{1}{P_{33}}[(P_{21}P_{32} - P_{22}P_{31})\bar{\phi}_1(s) - (P_{12}P_{31} - P_{11}P_{32})\bar{\phi}_2(s) + (P_{12}P_{21} - P_{11}P_{22})\bar{\phi}_3(s)]$$

and the function $\psi_0(s)$ is defined as

$$\psi_0(s) = \left(P_{11} - \frac{P_{31}}{P_{33}} P_{13}\right)\left(P_{21} - \frac{P_{32}}{P_{33}} P_{23}\right) - \left(P_{12} - \frac{P_{32}}{P_{33}} P_{13}\right)\left(P_{21} - \frac{P_{31}}{P_{33}} P_{23}\right). \tag{17}$$

In order to get the inverted expressions for these transfer potentials, Convolution theorem can be applied. The inverted form of the transfer potentials are

$$\theta_1 = 2 \sum_{n=1}^{\infty} \sum_{j=1}^{3} \frac{\mu_n}{\psi_n} L_j^n \cos \mu_n v_j x \exp(-\mu_n^2 Fo) \tag{18}$$

$$\theta_2 = \left(\frac{2}{\varepsilon Ko}\right) \sum_{n=1}^{\infty} \sum_{j=1}^{3} \frac{\mu_n}{\psi_n} L_j^n (1 - v_j^2) \cos \mu_n v_j x \exp(-\mu_n^2 Fo) \tag{19}$$

and

$$\theta_3 = \frac{2}{\varepsilon Bu} \sum_{n=1}^{\infty} \sum_{j=1}^{3} \frac{\mu_n}{\psi_n} L_j^n \sigma_j \cos \mu_n v_j x \exp(-\mu_n^2 Fo) \tag{20}$$

where

$$L_1^n = \int_0^{Fo} \left[\left(P_{22}^n - \frac{P_{32}^n}{P_{33}^n} P_{23}^n\right)\phi_1(u) - \left(P_{12}^n - \frac{P_{32}^n}{P_{33}^n} P_{13}^n\right)\phi_2(u) + \frac{1}{P_{33}^n}(P_{12}^n P_{23}^n - P_{22}^n P_{13}^n)\phi_3(u)\right] \exp(\mu_n^2 u)\, du$$

$$-L_2^n = \int_0^{Fo} \left[\left(P_{21}^n - \frac{P_{31}^n}{P_{33}^n} P_{23}^n\right)\phi_1(u) - \left(P_{11}^n - \frac{P_{31}^n}{P_{33}^n} P_{13}^n\right)\phi_2(u) + \frac{1}{P_{33}^n}(P_{21}^n P_{13}^n - P_{11}^n P_{23}^n)\phi_3(u)\right] \exp(\mu_n^2 u)\, du$$

$$L_3^n = \int_0^{Fo} [(P_{21}^n P_{32}^n - P_{22}^n P_{31}^n)\phi_1(u) - (P_{12}^n P_{31}^n - P_{11}^n P_{32}^n)\phi_2(u) + (P_{12}^n P_{21}^n - P_{11}^n P_{22}^n)\phi_3(u)] \frac{1}{P_{33}^n} \exp(\mu_n^2 u)\, du$$

$$P_{1j}^n = \left(A_1 + \frac{1 - v_j^2}{\varepsilon Ko}\right) \cos \mu_n v_j - \mu_n v_j \sin \mu_n v_j$$

$$P_{2j}^n = \frac{1 - v_j^2}{\varepsilon Ko} B_2 \cos \mu_n v_j + \left(\frac{1 - v_j^2}{\varepsilon Ko} + A_2 + \frac{\sigma_j C_1}{\varepsilon Bu}\right) \mu_n v_j \sin \mu_n v_j$$

$$P_{3j}^n = \frac{\sigma_j}{\varepsilon Bu} \cos \mu_n v_j.$$

The quantities μ_n are the characteristic roots, determined by the equation

$$\left(P_{11}^n - \frac{P_{31}^n}{P_{33}^n} P_{13}^n\right)\left(P_{22}^n - \frac{P_{32}^n}{P_{33}^n} P_{23}^n\right) - \left(P_{21}^n - \frac{P_{31}^n}{P_{33}^n} P_{23}^n\right)\left(P_{12}^n - \frac{P_{32}^n}{P_{33}^n} P_{13}^n\right) = 0 \tag{21}$$

and

$$\begin{aligned}
\psi_n = &\left[v_1 Q_{11}^n - \frac{P_{31}^n}{P_{33}^n}(v_3 Q_{13}^n + Q_{31}^n P_{13}^n)\right]\left[P_{22}^n - \frac{P_{32}^n}{P_{33}^n} P_{23}^n\right] \\
&- \left[v_2 Q_{22}^n - \frac{P_{32}^n}{P_{33}^n}(v_3 Q_{23}^n + Q_{32}^n P_{23}^n)\right]\left[P_{11}^n - \frac{P_{31}^n}{P_{33}^n} P_{13}^n\right] \\
&+ \left[v_1 Q_{21}^n - \frac{P_{31}^n}{P_{33}^n}(v_3 Q_{23}^n + Q_{31}^n P_{23}^n)\right]\left[P_{12}^n - \frac{P_{32}^n}{P_{33}^n} P_{13}^n\right] \\
&- \left[v_2 Q_{12}^n - \frac{P_{32}^n}{P_{33}^n}(v_3 Q_{13}^n + Q_{32}^n P_{13}^n)\right]\left[P_{21}^n - \frac{P_{31}^n}{P_{33}^n} P_{23}^n\right]
\end{aligned} \tag{22}$$

where

$$Q_{1j}^{n} = \mu_n v_j \cos \mu_n v_j + \left(A_1 + \frac{1-v_j^2}{\varepsilon Ko} B_1 + 1 \right) \sin \mu_n v_j \tag{23}$$

$$Q_{2j}^{n} = (1-v_j^2)\frac{B_2}{\varepsilon Ko} \sin \mu_n v_j + \left(\frac{1-v_j^2}{\varepsilon Ko} + A_2 + \frac{\sigma_j C_1}{\varepsilon Bu} \right)(\mu_n v_j \cos \mu_n v_j + \sin \mu_n v_j) \tag{24}$$

and

$$Q_{3j}^{n} = v_j \tan \mu_n v_j - v_3 \tan \mu_n v_3. \tag{25}$$

From expressions (18)–(20) it is observed that the transfer potentials are influenced by each of the surface fluxes.

ANALYSIS

In the absence of reliable experimental data, it is not possible to check the result obtained, however, numerical calculations are performed to study the kinetics of drying mechanism. In doing so, we will confine ourselves only by prescribing the surface fluxes ϕ_j in the form of constants Ki_j. The following reasonable values of the thermophysical coefficients and the criteria are selected:

$A_1 = 10$,	$A_2 = 0.25$,	$B_1 = 1.134$,	$B_2 = 20$
$C_1 = 0.185$,	$Ki_1 = 21.34$,	$Ki_2 = 20$,	$Ki_3 = 0$
$Lu = 0.3$,	$\varepsilon = 0.7$,	$Ko = 9.0$,	$Pn = 0.25$
$Lu_p = 500$,	$Bu = 10^{-3}$.		

Expressions (18)–(20) contain an exponentially decreasing series in Fo which converges rapidly with the increase of the generalised time, Fo and for a certain value of Fo (say $Fo \geqslant 0.7$) the first term of the series is sufficient to get the accurate result. The series expressions of equations (18)–(20) are split for the first term only. The first value of the characteristic root μ_n is calculated by solving equation (21) by Newton's approximation method which is used in evaluating the expressions of the transfer potentials.

Figures 1 and 2 describe the distributions of temperature and moisture inside the body for different values of the generalised time, Fo. From the figures we see that the moisture content of the surfaces of the material are smaller than those of the internal ones and the centre has the maximum value of the noisture content. The reverse takes place in case of temperature distributions of the body. The surface layer has the greater temperature than those of the internal ones. This causes a rapid transfer of moisture from the centre towards the surface. The evaporation from the surface also takes place very rapidly. This intensifies the process of drying material.

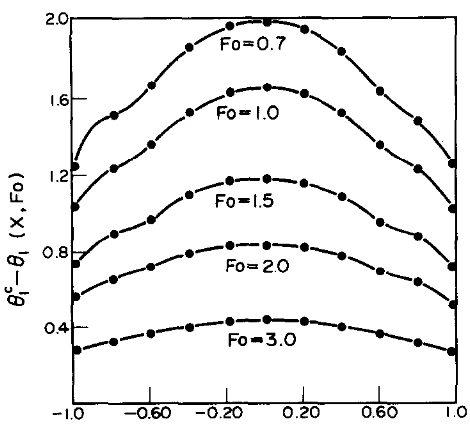

FIG. 1. Temperature distributions inside the body.

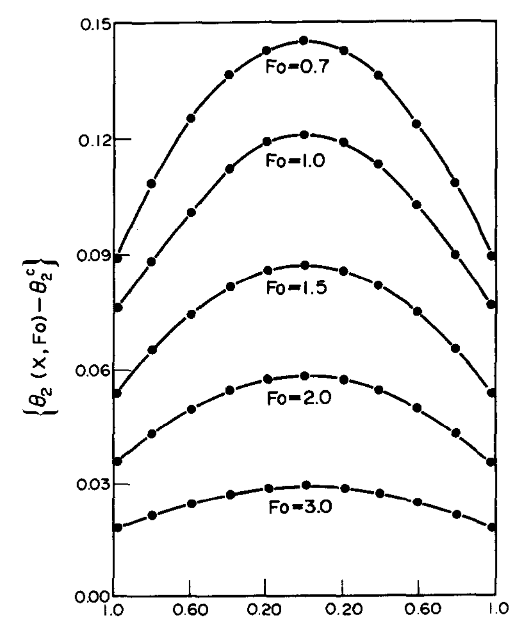

FIG. 2. Moisture transfer inside the body.

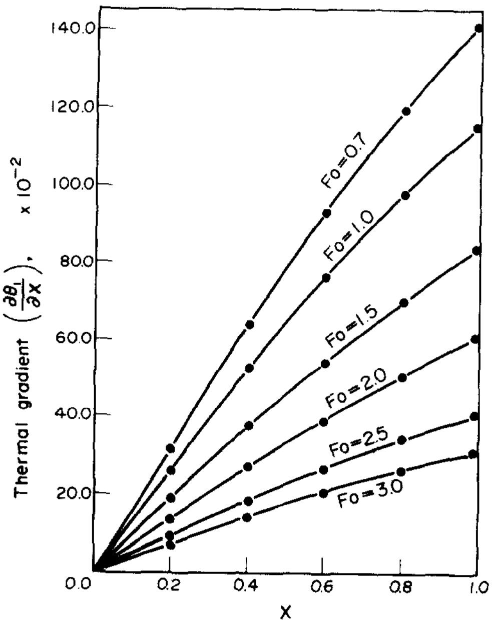

FIG. 3(a). Thermal gradient inside the body.

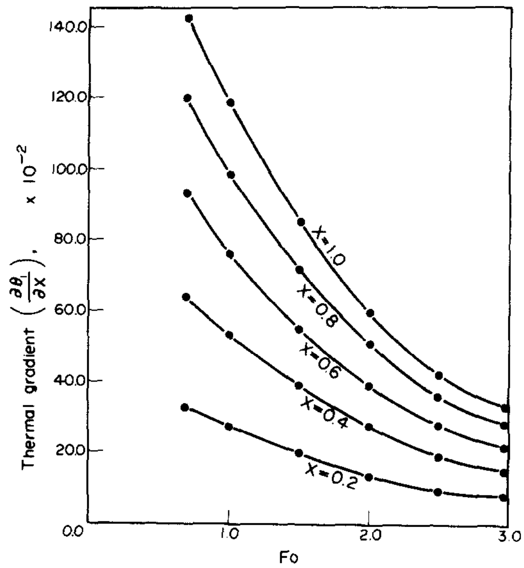

FIG. 4(a). Variation of thermal gradient with *Fo*.

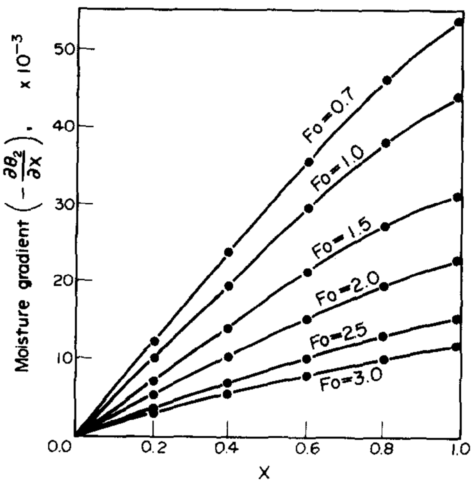

FIG. 3(b). Moisture gradient inside the body.

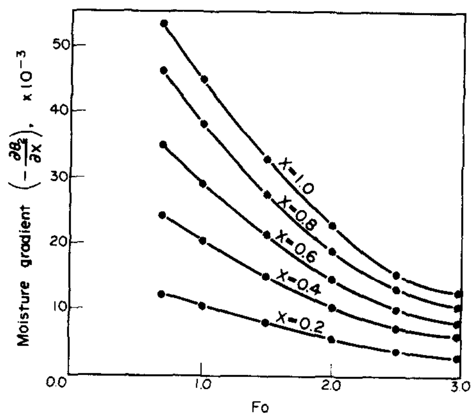

FIG. 4(b). Variation of moisture gradient with *Fo*.

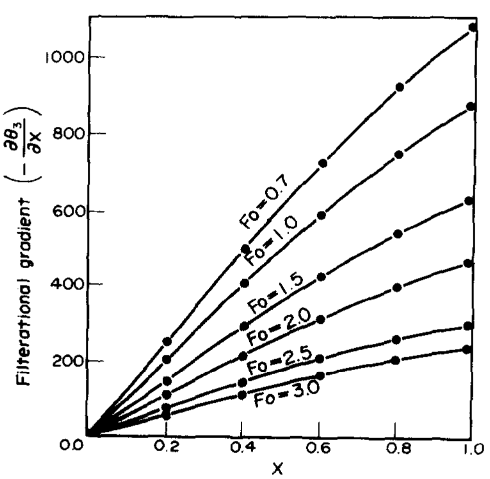

FIG. 3(c). Filterational gradient inside the body.

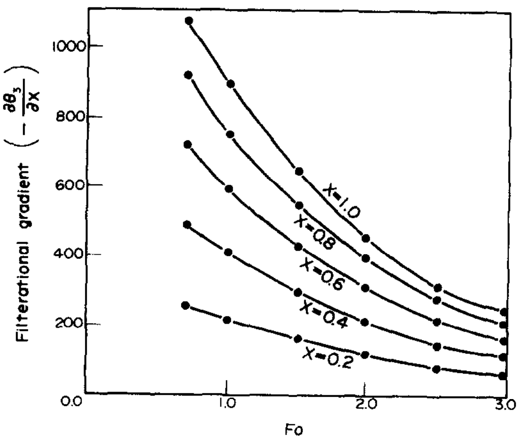

FIG. 4(c). Variation of filterational gradient with *Fo*.

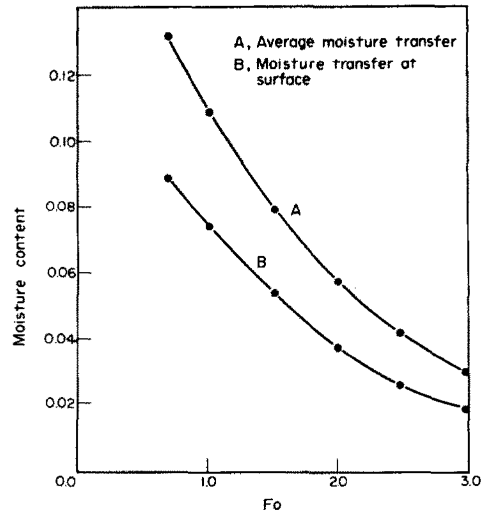

FIG. 5. A comparison of moisture transfer from the surface with the average moisture transfer of the body.

Figures 3 and 4 show the variation of the gradients of heat, matter and filtration in different parts of the body at different generalised times. The curves for the gradients of matter and filtration are symmetrical and also these gradients have negative signs which indicate a lack of agreement between the direction of the moisture flow vector and the direction of the gradients, and correspond to the transfer of moisture from the central layers towards the surface. Further, the filtrational gradient favours the molar motion of the vapour from centre to the surface. The temperature gradient has the positive sign which shows that the flow of heat occurs from the surface towards the centre. Further, the fluxes of the layers nearer to the surface fall rapidly with the generalised time and as it increases, an almost steady state is arrived at. The results are in good accordance with the results of Lebedev [8].

Figure 5 gives a comparative study of the moisture transfer of the body. This shows that during the period of drying the average of the moisture is transferred more rapidly than the transfer of moisture from the surface.

CONCLUSION

The kinetics of drying mechanism is investigated for the capillary porous body. It is observed that the process is intensified by the filtrational drying.

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SECHAGE INTENSIF D'UNE PLAQUE INFINIE

Résumé—On étudie la cinétique du mécanisme de séchage dans un corps poreux capillaire. Les valeurs locales de la température, du taux d'humidité et de la pression sont déterminées analytiquement sous des conditions aux limites de type le plus général. La nouveauté dans ce travail réside dans l'introduction de conditions généralisées à la surface qui incluent tous les types de loi d'intéraction entre le corps humide et son voisinage. Les résultats analytiques indiquent que le processus est intensifié par le séchage avec filtration.

INTENSIVE TROCKNUNG EINER UNENDLICH AUSGEDEHNTEN PLATTE

Zusammenfassung—Es wird die Kinetik des Trocknungsmechanismus für den Fall eines kapillar-porösen Körpers untersucht. Lokale Temperaturen, Feuchtigkeitsgehalte und Drücke werden für den allgemeinsten Fall von Randbedingungen analytisch ermittelt. Die Neuheit der Arbeit liegt in der Betrachtung verallgemeinerter Oberflächenbedingungen, welche alle Arten der gegenseitigen Beeinflussung zwischen dem feuchten Körper und seiner Umgebung einschließen. Das analytisch ermittelte Ergebnis deutet an, daß der Prozeß durch die Filtrationstrocknung intensiviert wird.

ИНТЕНСИВНАЯ СУШКА ПЛАСТИНЫ БЕСКОНЕЧНЫХ РАЗМЕРОВ

Аннотация — Исследуется кинетика процесса сушки капиллярно-пористого тела. Дается аналитический расчет локальных значений температуры, влагосодержания и давления при самых общих граничных условиях. Новизна работы состоит в рассмотрении обобщенных условий на поверхности, учитывающих все закономерности взаимодействия влажного тела с окружающей средой. Теоретический результат указывает на интенсификацию процесса в случае фильтрационной сушки.